Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

R. D. BOURKE

FLUX PENETRATION IN THIN WALLED SUPERCONDUCTING CYLINDERS

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Department of Aeronautics and Astronautics
Stanford University
Stanford, California

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SUMMARY

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Flux penetration into thin walled superconducting cylinders subjected to a uniform longitudinal field is investigated. A review of past work on the subject is given including predictions that penetration will occur at fields considerably less than critical. A detailed analysis of the field distribution is made for the geometry of interest and an attempt is made to observe surface breakdown for cylinders of various wall thicknesses. A method for generating continuous magnetization curves and therefore observing the onset of flux penetration is described. Contrary to some past thoughts on the subject, the onset of penetration is shown to occur at values of the surface field near critical. Finally these results are discussed in relation to recent work on the cryogenic gyro. Author

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1. INTRODUCTION

The recent success of a cyrogenic gyroscope (Refs. 1, 2) revives the interest in the use of a hollow superconducting sphere as a gyro rotor. This arrangement has the advantage that relatively high accelerations can be sustained without exceeding the critical field of the surface. It has been suggested that a high-quality spherical rotor could be constructed by simply plating a suitable superconductor on some low-density spherical substrate (Ref. 3).

Independent of the work on the cryogenic gyro, the problem of the magnetic field about a superconducting sphere has been thoroughly investigated both theoretically and experimentally. That of the field about a cylinder has been treated less extensively. In particular, classical books on the subject (e.g. London, Ref. 4) discuss two interesting but physically unrealizable cases: infinitely long cylinders with their axes parallel or transverse to a uniform applied field. The results are well known; in the first case the field penetrates when its nominal value reaches the critical field for the specimen, and in the second it penetrates at half that level.

Classically this initial penetration has been assumed to be independent of the material beneath the superconducting surface. Some authors, however, have contested this (Refs. 5-8). Serin, Gittleman and Lynton (Refs. 4, 5) have investigated hollow cylinders with plane ends in transverse fields. Their thermodynamic analysis indicates that these cylinders have lower free energies in the normal rather than the superconducting state at fields less than critical if the ratio of inner to outer diameters is above a certain value. They seem to infer from these results that cylinders with sufficiently thin walls spontaneously go over into the normal state even though the field on the surface has nowhere reached critical. This appears to be experimentally confirmed by their results.

Oakes (Ref. 7) has stated that a consequence of the thermodynamic analysis is that the superstate is metastable at best; that is, it can be at a local energetic minimum but there is an absolute minimum elsewhere. He points out, however, that if the applied field is always less than $(1 - \eta)H_c$ (where η is the demagnitization factor in the direction of interest) there is no obvious mechanism for precipitating the normal state. It should be noted that the true value of η in Ref. 5 was unknown; no quantitative analysis of the effects of the plane ends seems to have been made. Oakes therefore suggests that it might be instructive to study a thin walled body whose true demagnetization coefficient is well established and that the results might be applicable to the cryogenic gyro. It is the purpose of this paper to present the results of such a study.

2. MAGNETIC FIELD DISTRIBUTION

The specimens studied were copper cylinders, electroplated with lead and approximately 5 cm. long and 1 cm. in diameter with hemispherical ends. They were placed in a uniform field parallel to their axes. The magnitude of the field on the surface of the cylinders was calculated (assuming they are perfect diamagnets) by the hydrodynamic method of Smith and Pierce (Ref. 9) as applied to the superconductive case in Ref. 10. A plot of the field strength on the body surface is shown in Fig. 1 with the magnitude of the tangential field plotted in the normal direction. Note that the maximum field occurs on the hemispherical end (as one would expect) and is 1.36 times the nominal field. Since the demagnetization factor is just a measure of the flux concentration due to the distortion of the field by the sample, it may be calculated from the relation $H_{max} = H_{nominal}/(1 - \eta)$. This implies a demagnetization factor of .264, quite different from the value of zero usually considered for an infinite cylinder in a longitudinal field.

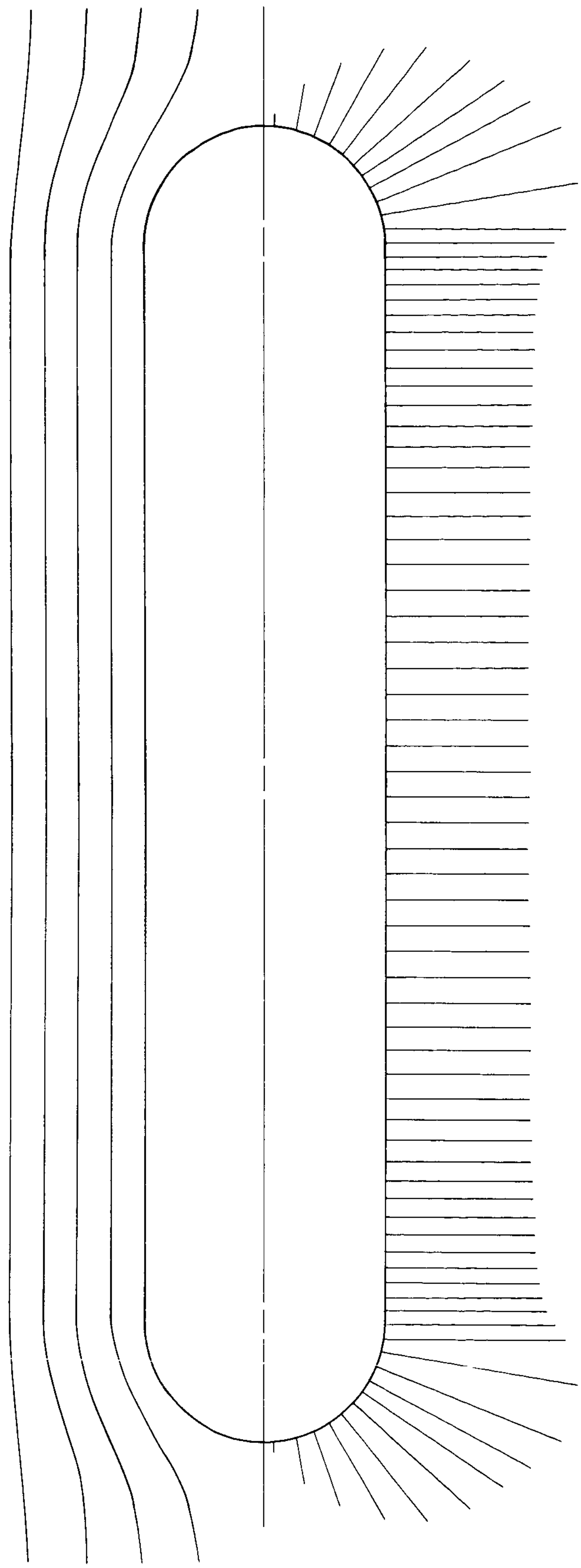

FIGURE 1

Tangential Field Strength Plotted in
the Normal Directions and Approximate Flux Lines

3. EXPERIMENTAL DETAILS

A simple method often used for testing the magnetization and therefore the flux penetration into superconducting samples operates as follows. A matched pair of oppositely wound series coils are arranged coaxially with the sample in one as shown in Fig. 2. When the sample's magnetization is zero there is no net flux threading the single series wound coil and a displacement of the sample or a change of the applied field will not produce a voltage at its terminals. If, on the other hand, the sample is acting like a diamagnet the net flux in the coils will be non zero and changing the applied field or moving the sample will induce a voltage. Ordinarily the applied field is set and the sample is moved from one coil to the other switching the net flux and inducing a voltage at the terminals. The output is read on a ballistic galvanometer (essentially an integrating voltmeter).

The method used for the present studies was a variation of this technique. Note that if the applied fields are initially zero then an application of the field will induce a voltage, the integral of which will be proportional to the net flux in the coil pair and consequently a measure of the sample's flux exclusion or negative magnetization.

The integration in this case was carried out by an Electronics Associates TR-20 analog computer with an amplification of one hundred. Figure 3 shows diagrammatically the wiring arrangement. It was necessary to sum a bias voltage into the integrator to counteract DC offsets in the amplifiers. Note that because of the availability of a large number of integrators, it was possible to test several samples simultaneously. The integrated signal was fed to one of the vertical deflection plates of a multichannel oscilloscope. The horizontal deflection was proportional to the applied field as measured by the magnet current.

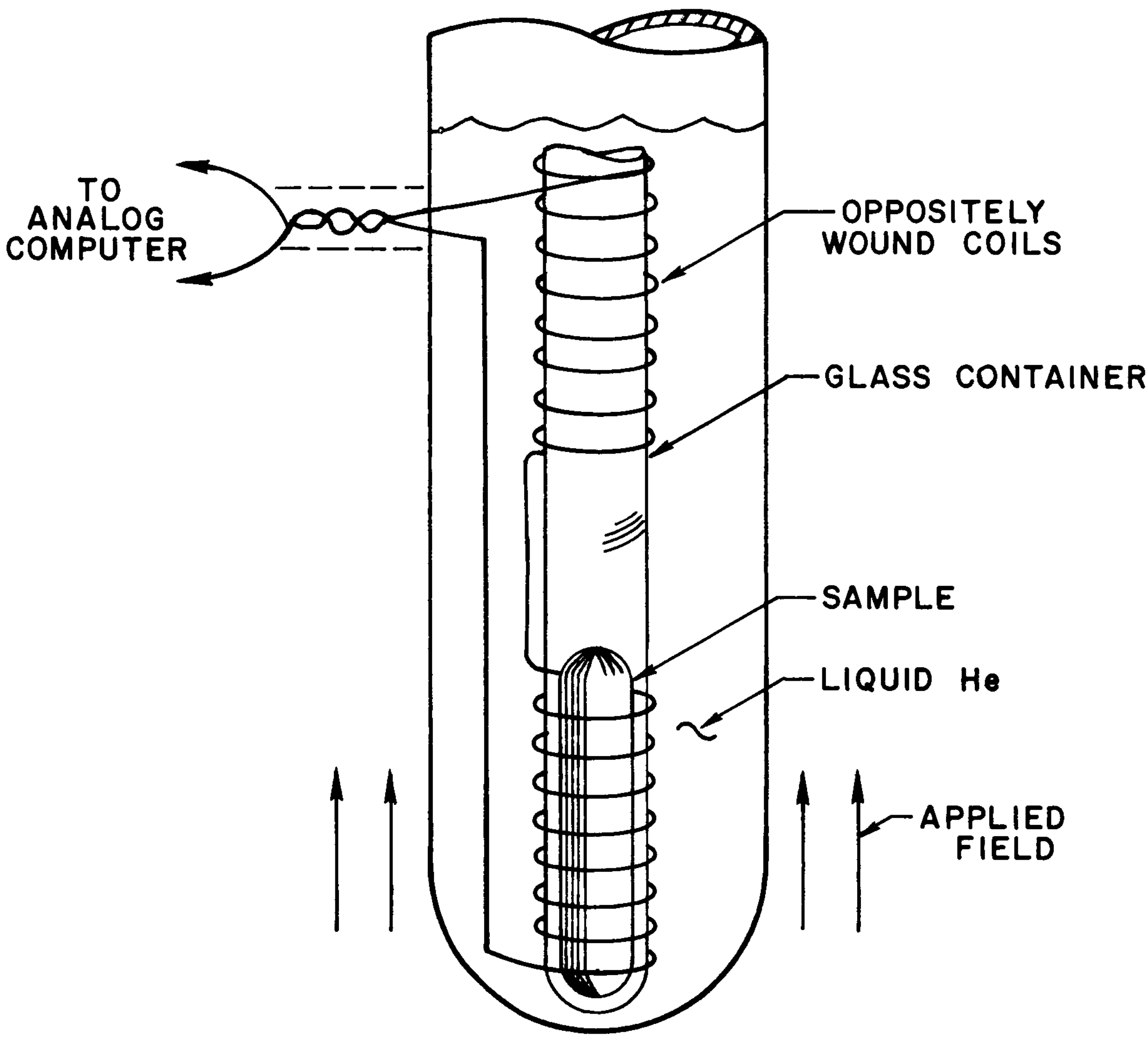

FIGURE 2

Magnetization Apparatus

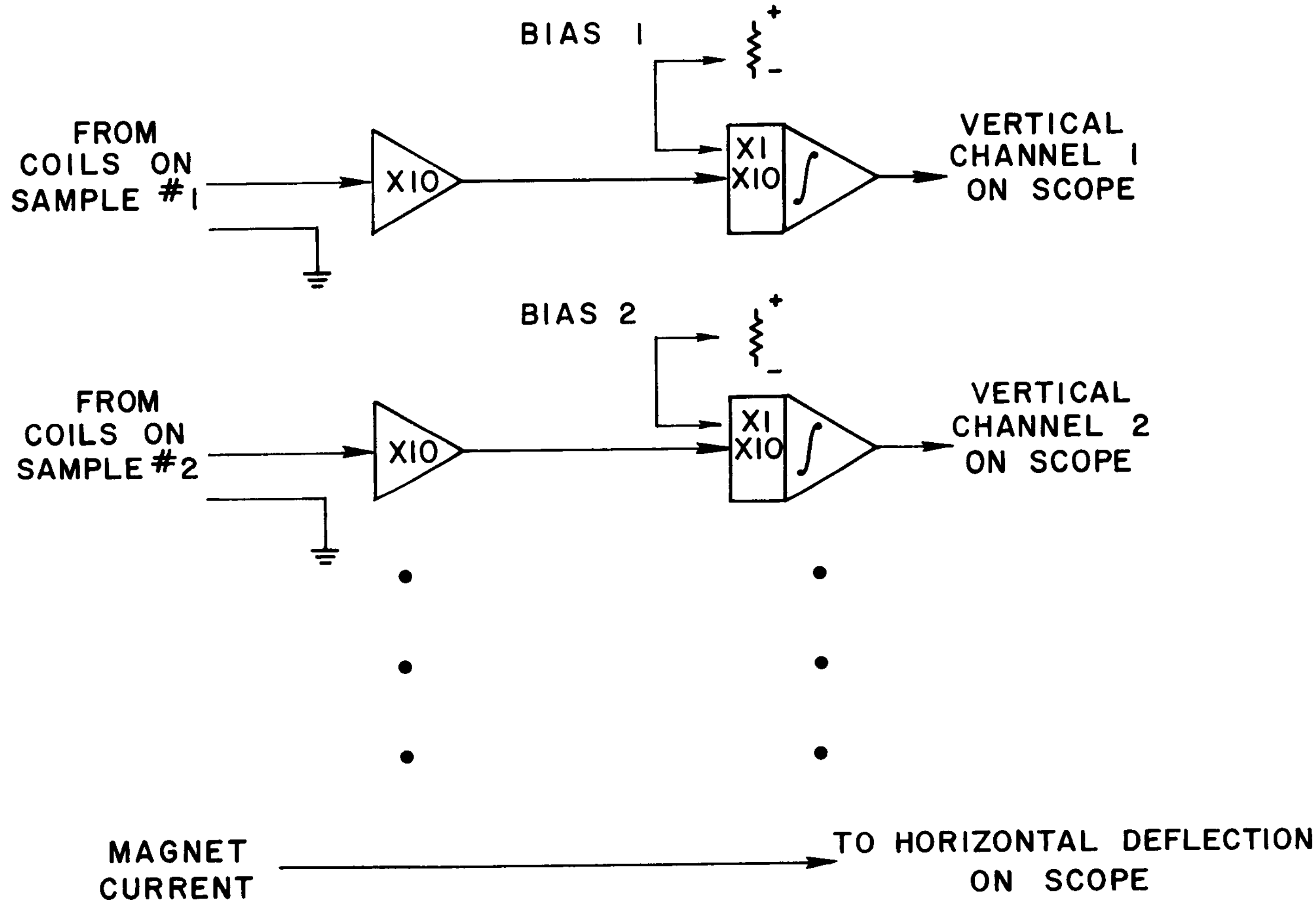

FIGURE 3

Analog Computer Circuitry

The advantages of the present method are several. Most important, it affords a method for generating continuous magnetization curves rather than testing at discrete points. Second, it eliminates the mechanical apparatus necessary for moving the sample. Because analog computers have many integrators, and scopes with multitraces are readily available, this method allows testing of several samples simultaneously. In short, it represents a considerable gain in efficiency over the older method without an appreciable loss in accuracy.

The samples themselves consist of polished copper rods with lead in thicknesses between 15 and 100 microns electroplated onto the surface. The plating was done by the Aircraft Processing Co. of Belmont, California, using a standard lead fluo-borate bath. Samples were plated simultaneously in the same bath with the only variation being the duration of emersion, or equivalently the thickness of the covering. The thickness was determined by measuring the cylinder diameter before and after the plating. Two sets of samples were used and they were processed at different times. It is therefore possible that the superconductive properties of the lead in the two sets could differ but there should be no variation within the sets themselves.

4. EXPERIMENTAL RESULTS

A simultaneous plot of the magnetization of four of the samples of different thickness is shown in Fig. 4. Note that shortly after deviating from linearity the curve plunges rapidly to zero indicating complete penetration. The lower portion of the curve in each case is the return or removal of the applied field. Note that nearly all the penetrating flux is trapped and there is no evidence whatsoever of the Meissner effect just as one would expect from the small fraction of the total volume that is actually superconducting.

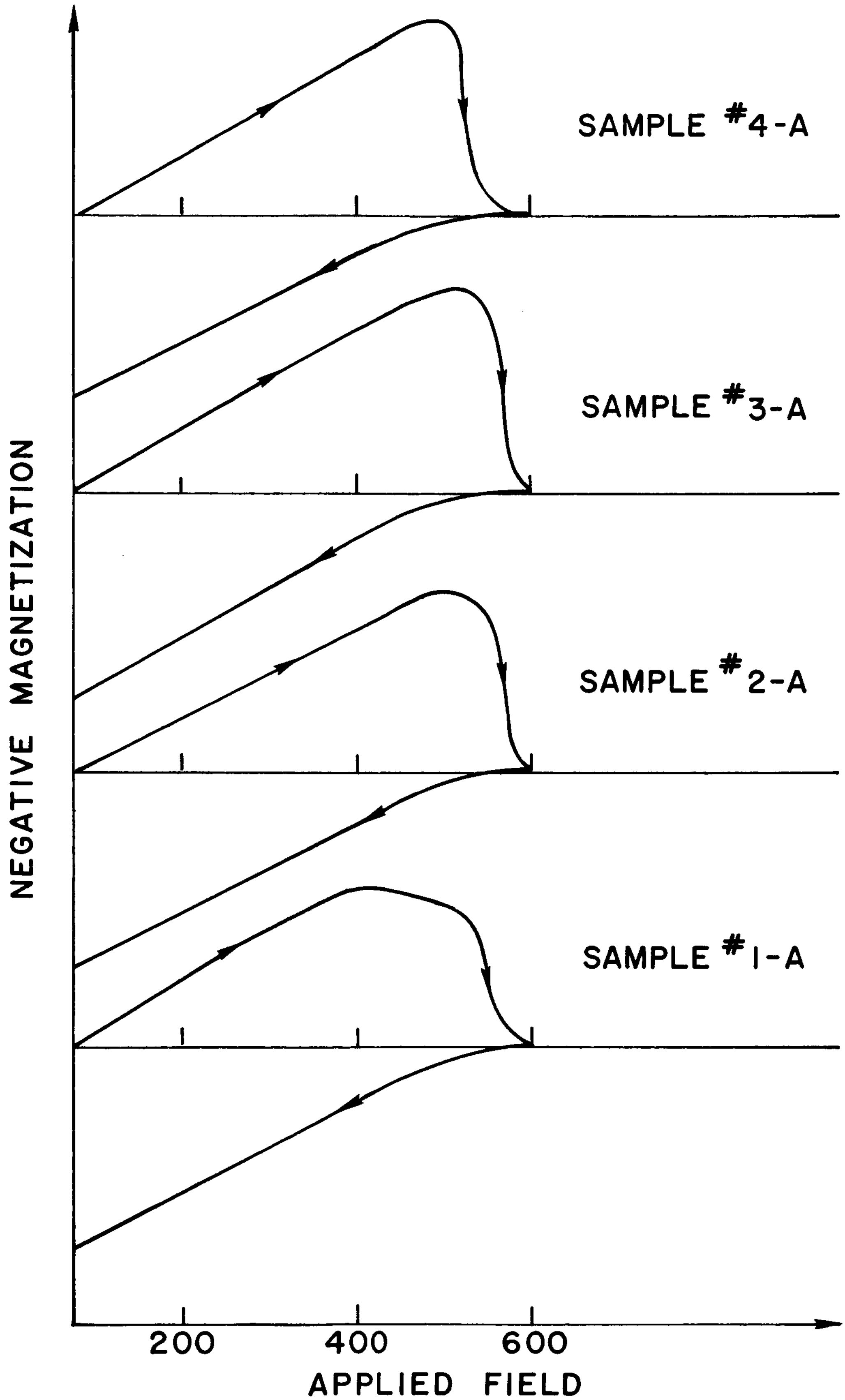

FIGURE 4

Simultaneous Magnetization Tests of Four Samples

Table I gives the sample thickness, external field for the onset of penetration as indicated by the magnetization curves' deviation from linearity, the maximum surface field using the calculated demagnetization factor, and the field for complete penetration (magnetization less than 5% of its maximum value) all at 4.2°K. It should be noted that experiments with very pure lead show a critical field of about 540 gauss at this temperature.

TABLE I

Penetration Values at 4.2°K

Sample and Set	Pb Thickness	Applied Field at Onset of Penetration	Calculated Maximum Surface Field at Onset of Penetration	Applied Field for Complete Penetration
1 - B	15 microns	400 gauss	540 gauss	450 gauss
1 - A	15	390	520	530
2 - B	25	430	580	510
2 - A	38	450	610	560
3 - A	81	440	590	540
4 - A	112	460	620	530

5. DISCUSSION

In every case the thickness of the lead was considerably greater than the London penetration depth (≈.04 microns) so that it should display bulk properties as opposed to thin film properties. It therefore seems correct that if a thermodynamic argument like that of Refs. 4-7 is valid, it should apply to this case. No detailed thermodynamic calculation has been made for the geometry used here, but the rather thin walls seem to imply that the normal state would have a lower free energy than the superstate at fields considerably below critical. Regardless of this energy relation, one can say from noting the data that penetration is not occurring at fields catastrophically below the critical. As long as the field on the surface is uniformly below critical, it seems that there is no mechanism for precipitating flux penetration and the transition to the lower energy state. Therefore, in answer to the question Oakes has raised on the subject, Serin, Lynton and Gittleman have obtained their results primarily due to the unfavorable demagnetization factor (essentially one) which their sample possessed. Furthermore, the cryogenic gyro, when designed to avoid such demagnetization factors, should not be adversely affected by this phenomenon.

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